A Mesh-free Particle-Based Model for Microscale Shrinkage Mechanisms of Food Materials in Drying Conditions

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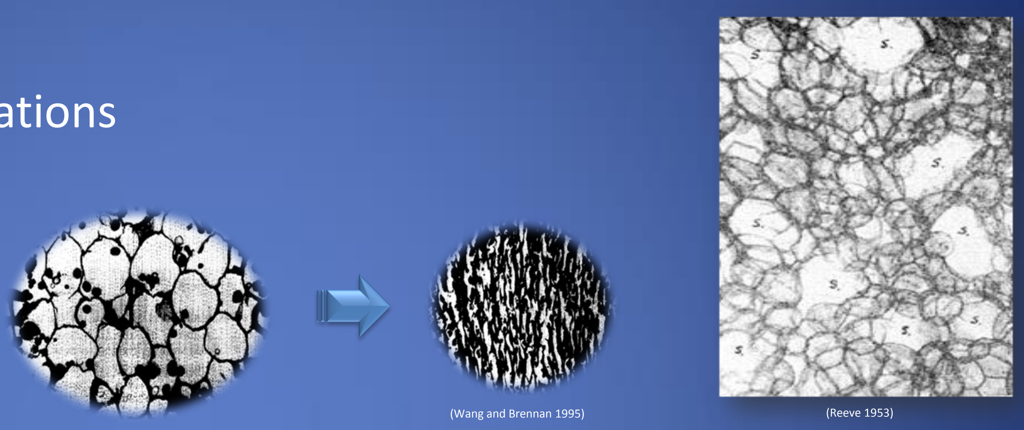
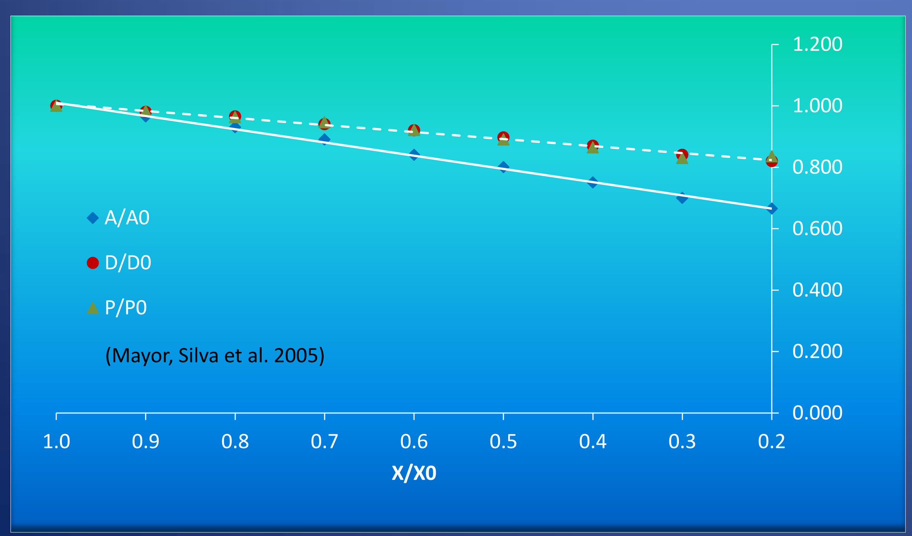
Background

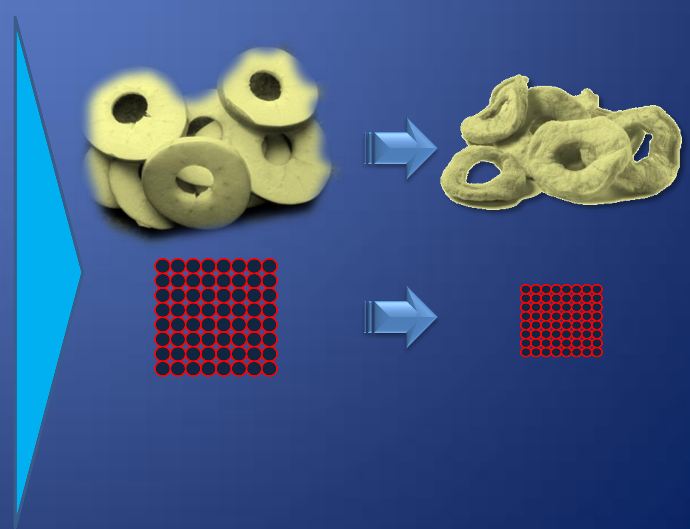
- Food drying is a significant industry

- Numerical modeling and simulations are essential for product/process improvements
- Problem domain:
  - Excessive deformations
  - Multi-phase
  - Discrete
  - Multi-scale
  - Phase change



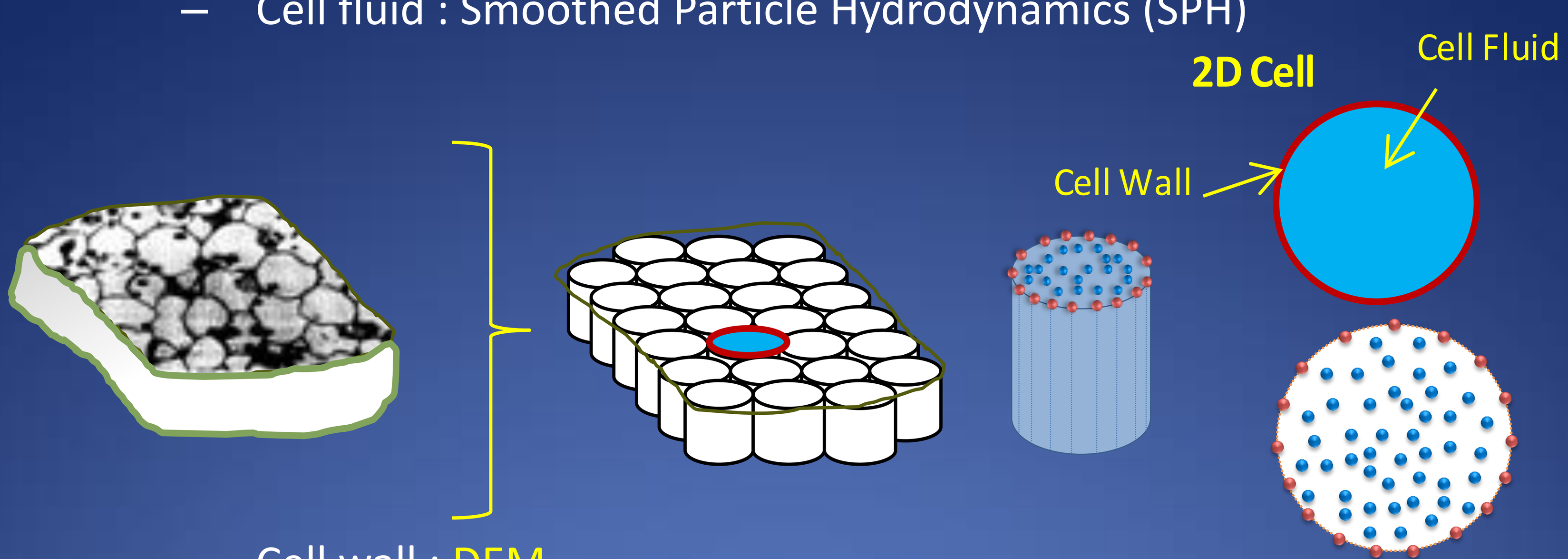
- No reliable software available for micro-structural modeling
- Experimental Findings: Direct relationship exists between moisture content and cellular deformations



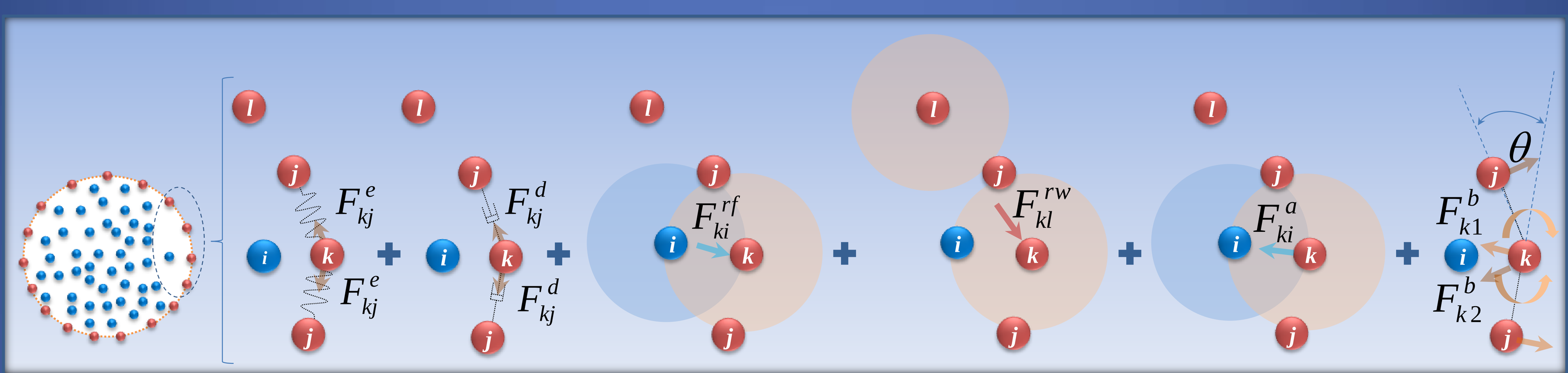


Methodology

- 2-D Single cell model
  - Cell wall : Discrete Element Method (DEM)
  - Cell fluid : Smoothed Particle Hydrodynamics (SPH)



- Cell wall : DEM



$$F_k = F^e{}_{kj} + F^d{}_{kj} + F^{rf}{}_{ki} + F^{rw}{}_{kl} + F^a{}_{ki} + F^b{}_k$$

$$F^e{}_{kj} = G z_0 t_0 \left(\lambda_\theta - \frac{1}{\alpha^2 \lambda_\theta^5}\right)$$

$$\alpha = \sqrt{\frac{\beta + \sqrt{\beta^2 - 4(\beta - 1)/\lambda_\theta^6}}{2}}$$

$$F^d{}_{kj} = -\gamma\, v_{kj}$$

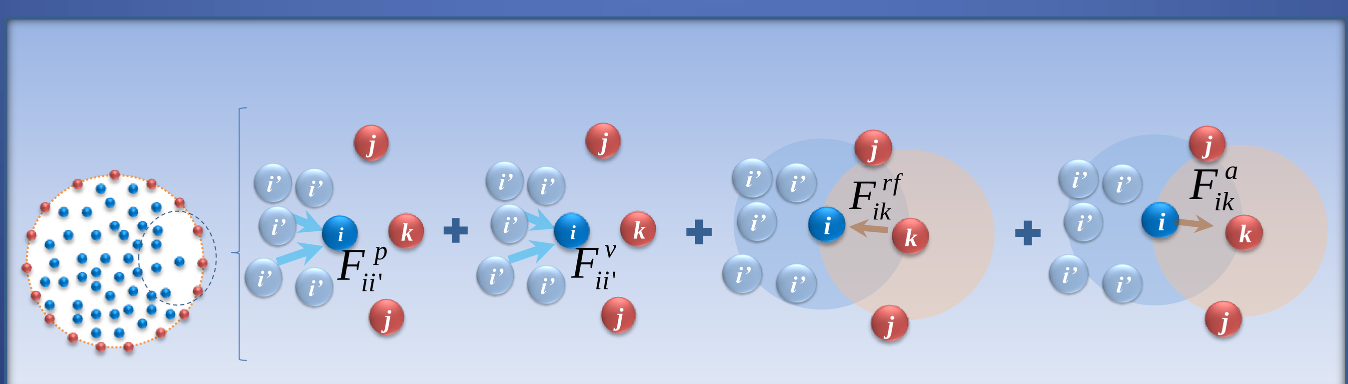
$$F^{rf}{}_{ki} = f^{rf}_{ki}\, x_{ki}$$

$$F^{rf}{}_{ki} = f^{rf}_{ki}\, x_{ki}$$

$$F^b{}_k = \frac{k_b}{l} \tan(\Delta\theta/2)$$

$$f^{rf}_{ki} = \begin{cases} f^{rf}_0 \left[\left(\frac{r_0}{r_{ki}}\right)^8 - \left(\frac{r_0}{r_{ki}}\right)^4\right]\left(\frac{1}{r_{ki}{}^2}\right) & \left(\frac{r_0}{r_{ki}}\right) \geq 1 \\ 0 & \left(\frac{r_0}{r_{ki}}\right) < 1 \end{cases}$$

- Cell fluid : SPH



$$F_i = F^p{}_{ii'} + F^v{}_{ii'} + F^r{}_{ik} + F^a{}_{ik}$$

$$F^p{}_{ii'} = -m_i \sum_{i'} m_{i'} \left(\frac{P_i}{\rho_i^2} + \frac{P_{i'}}{\rho_{i'}^2}\right)\left(\frac{1}{z}\right) \nabla_i W_{ii'}$$

$$F^v{}_{ii'} = m_i \sum_{i'} \frac{m_{i'}(\mu_i + \mu_{i'}) v_{ii'}}{\rho_i \rho_{i'}} \left(\frac{1}{z}\right)\left(\frac{1}{r_{ii'}} \frac{\partial W_{ii'}}{\partial r_{ii'}}\right)$$

$$\frac{d\rho_i^*}{dt} = m_i \sum_{i'} v_{ii'} \cdot \nabla_i W_{ii'}$$

$$\frac{dz}{dt} = \frac{z_{(t=t_0)} - z_{(t=t_0+t)}}{\Delta t}$$

$$\frac{dm_i}{dt} = -\frac{A_c L_p \rho_i}{N_f}(P_i + \Pi)$$

$$W_{ij} = \frac{15}{7\pi h^2} \begin{cases} \left(\frac{2}{3} - \frac{9}{8}R^2 + \frac{19}{24}R^3 - \frac{5}{32}R^4\right) & 0 \leq R \leq 2 \\ 0 & R > 2 \end{cases}$$

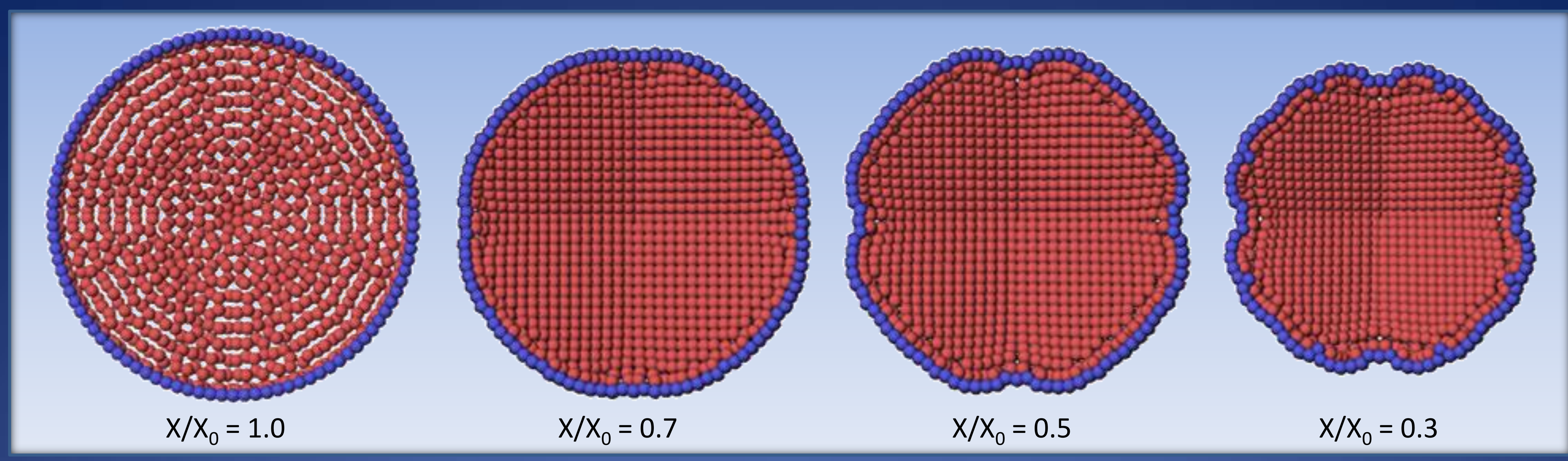
$$\frac{d\rho_i}{dt} = \frac{1}{z}\frac{d\rho_i^*}{dt} - \frac{\rho_i^*}{z^2}\frac{dz}{dt} + \frac{\rho_i}{m_i}\frac{dm_i}{dt}$$

$$P_i = P_0 + K\left[\left(\frac{\rho}{\rho_0}\right)^7 - 1\right]$$

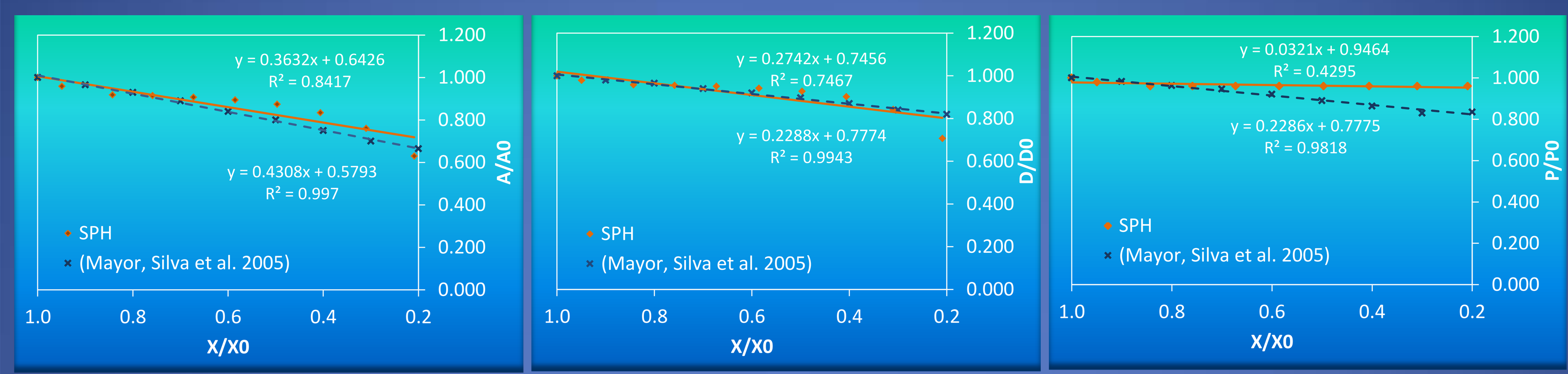
$$\rho_i^* = h\rho_i \qquad R = \frac{r_{ii'}}{h}$$

Results

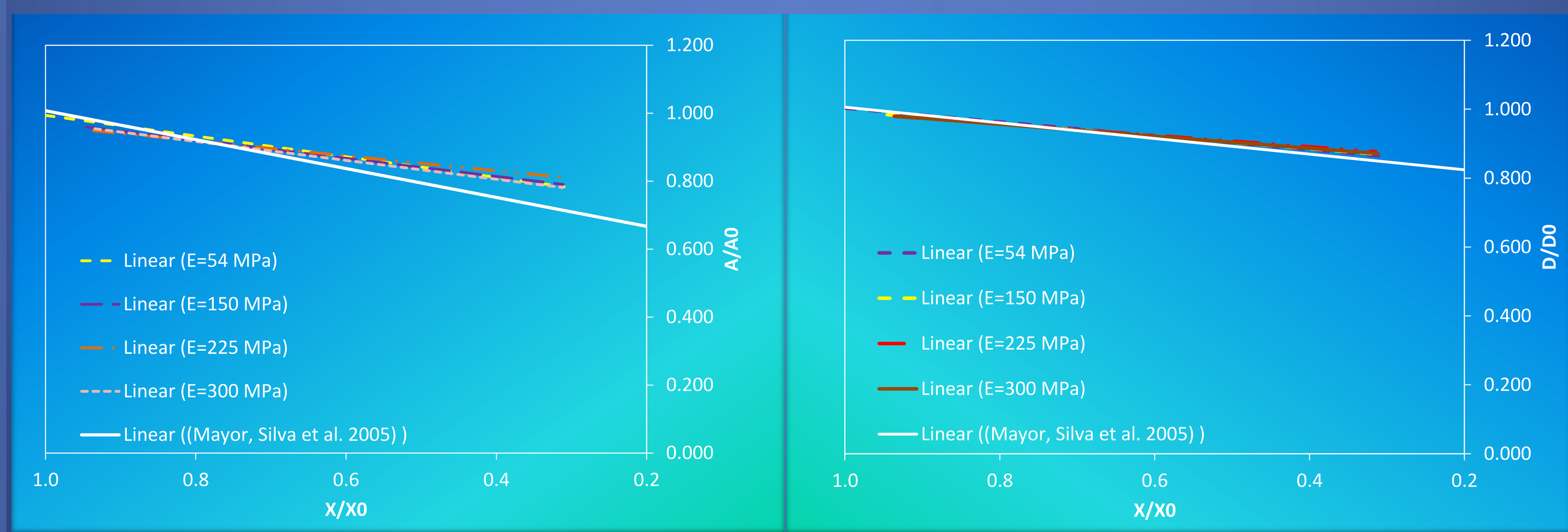
- Cellular deformations with change of moisture content (x/x_0)



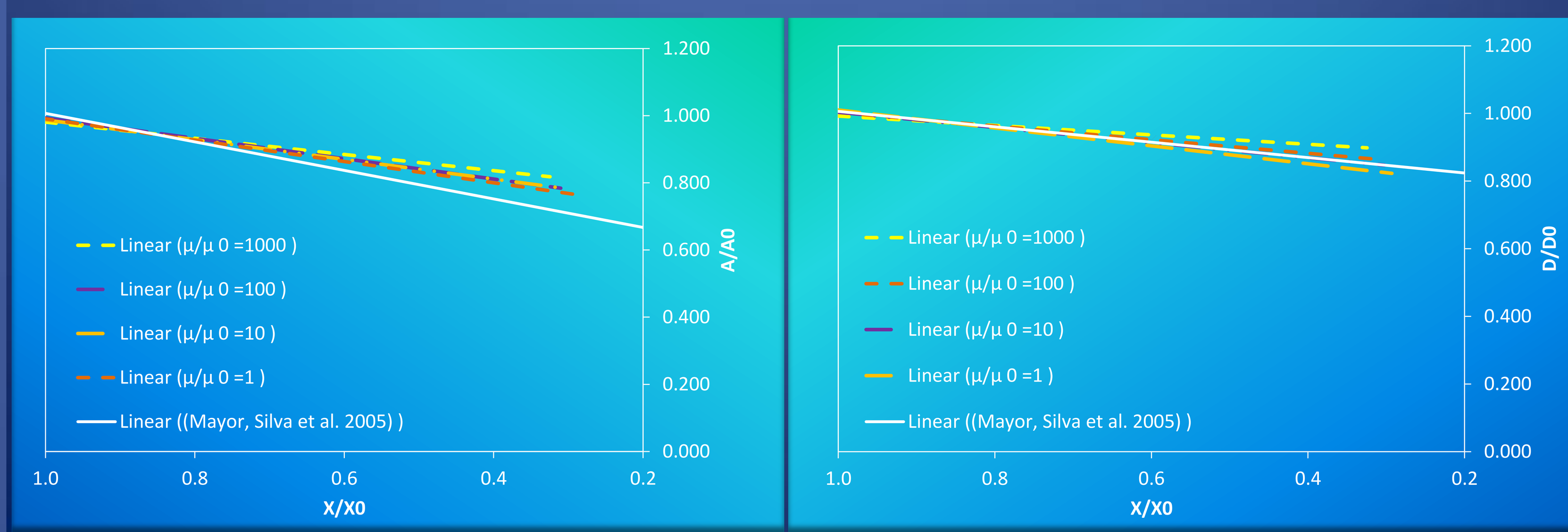
- Shrinkage predictions using normalized area (A/A_0), diameter (D/D_0) and perimeter (P/P_0)



- Model sensitivity to cell wall Young's modulus (E)



- Model sensitivity to cell fluid viscosity (μ)



Conclusion

- SPH-DEM approach: can promisingly be used to model cellular structural changes of food materials during drying conditions
- Shrinkage Predictions (Area & Diameter): good agreement with experiments
- Shrinkage Predictions (Perimeter): only shows some shrinkage at the first stage of drying until some positive turgor pressure exists
- Cell wall Young's modulus: influence on shrinkage is not so significant
- Cell fluid Viscosity: influence on shrinkage is significant; higher the viscosity higher the shrinkage; higher viscosity values provide better match with experimental findings

Outlook

- 2-D tissue model to study effects of intercellular space and middle lamella
- 3-D single cell model
- 3-D tissue model

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